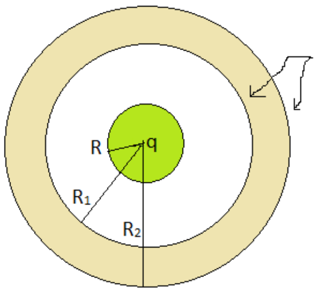
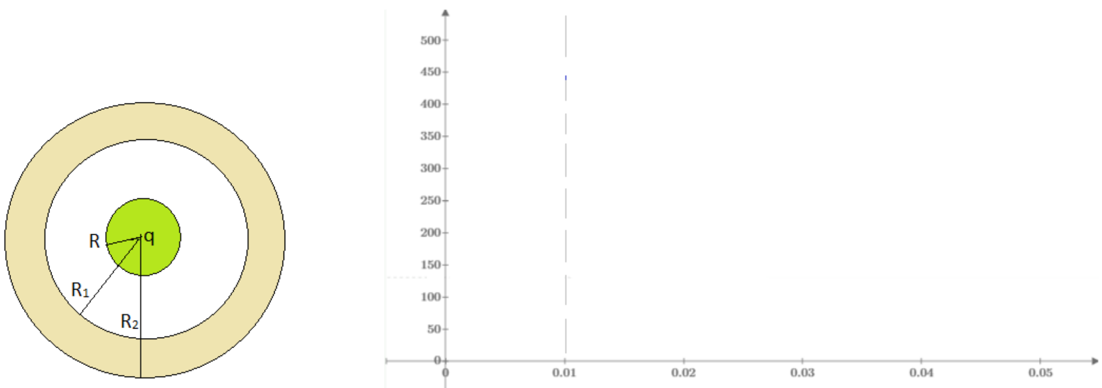
Homework 6: Electric Fields in Matter due 4/30

Problem 1. Say we have a spherical shell surrounding a charge q of radius R. Derive the following equation for the induced charge on the inner/outer surfaces of the shell.



$$q_{induced} = \frac{\chi_e}{1+\chi_e} q$$

Problem 2. Now with reference to the previous setup, suppose q = 5pC charge with a radius R = 1cm. Let R_1 = 2cm, and R_2 = 3cm. The empty space white space in between and outside is just air. If the shell is a perfect insulator (χ_e = 0), answer the following questions:



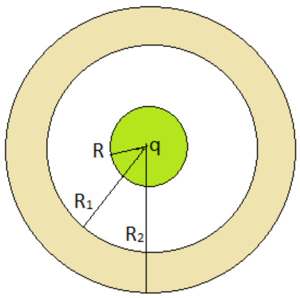
(a) Plot the electric field as a function of r, from r = 1cm to 5cm.

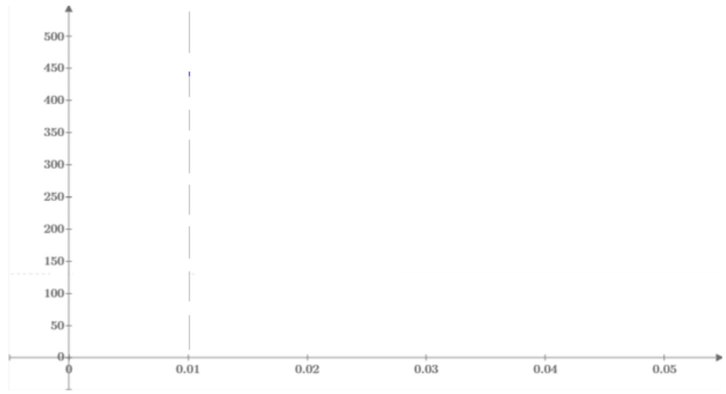
(b) Determine the induced charges on the surfaces, and specify their signs.

(c) Calculate the electric potential energy, 'elastic' potential energy, and total potential energy stored in the shell.

(d) What would be the dielectric strength of a perfect insulator?

Problem 3. Now let's fill the shell with pyrex (κ_e = 5.6).





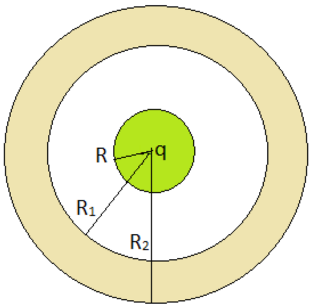
(a) Plot the electric field as a function of r, from r = 1cm to 5cm.

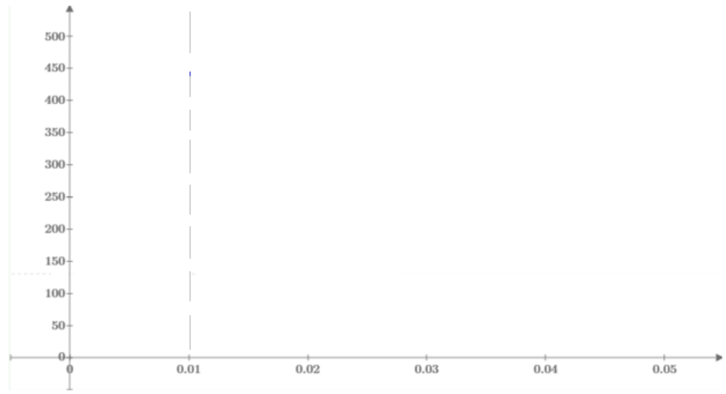
(b) Determine the induced charges on the surfaces, and specify their signs.

(c) Calculate the electric potential energy, 'elastic' potential energy, and total potential energy stored in the shell.

(d) The dielectric strength of pyrex is 14MN/C. What would our center q have to be to induce dielectric breakdown?

Problem 4. Now let's make the shell a perfect metal ($\kappa_e = \infty$).





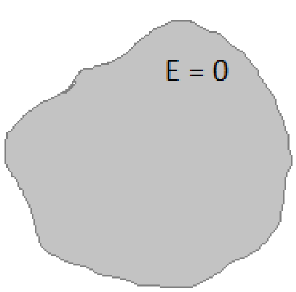
(a) Plot the electric field as a function of r, from r = 1cm to 5cm.

(b) Determine the induced charges on the surfaces, and specify their signs.

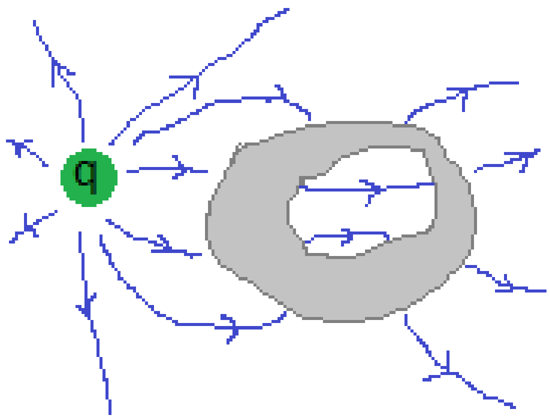
(c) Calculate the electric potential energy, 'elastic' potential energy, and total potential energy stored in the shell.

(d) What is the dielectric strength of a metal?

Problem 5. Use Gauss's law to show that as long as E = 0 inside a substance, like a metal for instance, there can be no *net* charge *within* that surface. I suggest you draw a Gaussian surface within the substance and see what happens, and your proof should take about three lines max.



Problem 6. Like my drawing? In problem 4, you should've seen that while E = 0 inside the metal, it still leaked through the metal into the air outside. But now consider the reverse. Say we have a charge outside the metal. Can the field lines leak into the cavity *inside* the metal? Prove (couple lines max) your answer one way or the other by considering the consequences of the fact that ΔV must be 0 around a closed loop. Note, this is the reason cell phone signals get lost in elevators, metal (Faraday) cages are used to shield sensitive electronics, and people wear tin foil hats to protect their brains from aliens' presumably EM probes.



Problem 7. Consider the following setups. Charge -q is deposited on a metal. And a neutral sphere hovers magically above it. Draw where the -q charges will distribute themselves in the metal, and where the induced charge, if any, will distribute itself in the sphere. Then draw a rough picture of what you expect the electric field lines to look like. On the left, assume the magic sphere is a perfect metal, and on the right assume the magic sphere is a perfect insulator.